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MULTIPLE-DECISION SELECTION AND RANKING PROCEDURES.(U)
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MULTIPLE-DECISION SELECTION AND RANKING PROCEDURES

FINAL REPORT ON CONTRACT DAAG29-80-C-0036

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by

Robert E. Bechhofer
Principal Investigator

July 31, 1981

U.S. ARMY RESEARCH OFFICE

CONTRACT DAAG29-80-C-0036

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FINAL REPORT ON CONTRACT DAAG29-80-C-0036

This is the final report on Contract DAAG29-80-C-0036 entitled "Multiple Decision Ranking and Selection Procedures." The report covers the period November 1, 1979 through July 31, 1981. The present contract with a budget of $20,000 was originally funded for the period November 1, 1979 through October 31, 1980; a no-new-funds extension was granted to continue research through July 31, 1981. The contract is a follow-up to Contract DAAG29-77-C-0003 (and earlier ones) which had similar missions. Because of the closeness of the objectives of these contracts, this report lists some technical reports and papers, the research for which was initated or completed under the earlier contract, but which was not completed or published until after the final report for that contract was submitted.

The following pages list the technical reports and published papers (and those accepted for or submitted for publication) which were written with the full or partial support of these contracts. The contents of most of these reports and papers have already been described in detail in the three Semi-Annual Progress Reports covering the periods August 16, 1978 - December 31, 1979, January 1, 1980 - June 30, 1980, and July 1, 1980 - December 31, 1980.

This report lists that

~~Most of the~~ papers deal with research on the subject of "Statistical multiple-decision ranking and selection procedures," a statistical methodology pioneered by the Principal Investigator and several colleagues; some of the papers deal with related statistical techniques. Interest in this subject continues to grow as is evidenced by the large number of published papers in this area, and by the appearance in the last few years of three textbooks devoted exclusively to this subject. The books are:

Selecting and Ordering Populations: A New Statistical Methodology (Wiley, 1977) by J. Gibbons, I. Olkin and M. Sobel, (569 pages).

Multiple Decision Procedures: Theory and Methodology of Selecting and Ranking Populations (Wiley, 1979) by S.S. Gupta and S. Panchapakesan, (573 pages).

Nonparametric Sequential Selection Procedures (Birkhaüser, 1980) by H. Buringer, H. Martin and K. - H. Schriever, (488 pages).

An invited review of the Gibbons-Olkin Sobel book by the Principal Investigator appeared in the Journal of the American Statistical Association, 75 (1980), 751-756.

The Principal Investigator, along with J. Gibbons (University of Alabama), S.S. Gupta (Purdue University) and I. Olkin (Stanford University), organized and presented the 1979 Annual Meeting Short Course on the subject of "Selecting and ordering populations" for the American Statistical Association. The course was held August 11 - 12, 1979 in Washington, D.C.; videotapes of the course should be available shortly from the ASA. The course was very well attended, and the topics aroused considerable interest on the part of the participants. A significant portion of the statistical methodologies which were presented and discussed represented the results of research by the Principal Investigator and his colleagues at Cornell University and elsewhere.

In addition, the Principal Investigator with Professor Shanti Gupta offered a 5-day seminar on "Selection and Ranking Procedures" sponsored by The George Washington University, School of Engineering and Applied Science and AMK Berlin in Berlin, Germany, November 17 - 21, 1980.

RECENT RESEARCH ON CONTRACT DAAG29-80-C-0036

The main thrust of new research on the present contract stems from a selection problem associated with quantal response curves; this problem has considerable practical importance. The statistical model underlying the quantal response curve problem was described in detail in the Semi-Annual Progress Report on this contract which covered the period January 1, 1980 - June 30, 1980. It was pointed out there that if certain assumptions hold for the $k \geq 2$ quantal response curves, then the selection problem for the k quantal response curves can be solved by solving an associated selection problem for $k \geq 2$ Bernoulli populations.

Recognition of the fact that a solution of the Bernoulli selection problem leads (under appropriate assumptions) to a solution of the quantal response selection problem suggested that intensive study of the former problem would be profitable. Such an investigation was undertaken by the Principal Investigator along with one of his Ph.D. students, Ms. Radhika Kulkarni. This study of the Bernoulli selection problem led to a major research breakthrough. Their findings are not only relevant to the quantile selection problem, but are also very important in their own right.

The first technical report on this subject, TR 510 entitled "Closed adaptive sequential procedures for selecting the best of $k \geq 2$ Bernoulli populations" has just been completed. Some of the results discussed in that report were presented at the Third Symposium on Statistical Decision Theory and Related Topics held at Purdue University, June 1 - 5, 1981 and will be published in the Proceedings of the Symposium. (Earlier presentations of portions of the paper were given at the spring regional meeting of the Biometric Society and American Statistical Association in Richmond, Virginia, March 23, 1981, and at the Annual Meeting of the Israeli

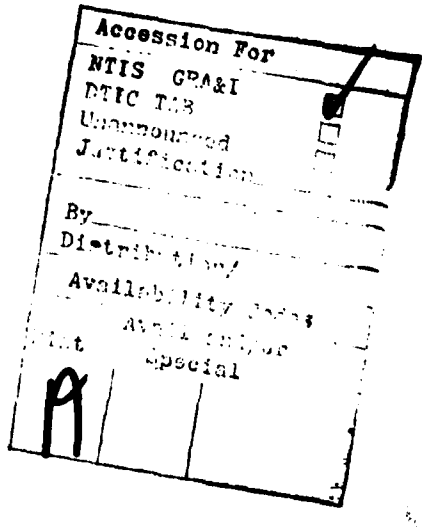
Statistical Society in Tel Aviv, Israel on May 28, 1981.) An abstract of the paper is given below. Additional papers on this subject are in preparation. A summary of some of the findings was also presented in a seminar at the Ballistic Research Laboratories, Aberdeen, on June 12, 1981. It is hoped that the studies that are in progress will form the basis for recommendations for new generalized sampling plans that might be incorporated in a Military Standard testing procedure for comparative Bernoulli trials. Also, as pointed out above, the results will be applicable to the quantal response curve selection problem.

Abstract of TR 510 (July 1981) entitled "Closed adaptive sequential procedures for selecting the best of $k \geq 2$ Bernoulli populations" by R.E. Bechhofer and R.V. Kulkarni.

"The goal of selecting that one of $k \geq 2$ Bernoulli populations which has the largest single-trial "success" probability $p_{[k]} = \max\{p_1,\ldots,p_k\}$ is treated. Consideration is restricted to procedures which take no more than n observations from any one of the k populations. One such procedure is the single-stage procedure of Sobel and Huyett [1957] which takes exactly n observations from each of the k populations. We propose a one-at-a-time adaptive sampling rule (R^*) which when used in conjunction with a particular stopping rule (S^*) and terminal decision rule (T^*) achieves the same probability of a correct selection as does the single-stage procedure uniformly in $\underset{\sim}{p} = (p_1,\ldots,p_k)$. Letting N denote the random total number of observations to terminate sampling using the procedure (R^*,S^*,T^*) we show that $n \leq N \leq kn-1$; for $p_{[k]} \to 0$ we have $P\{N = kn-1 \mid \underset{\sim}{p}\} \to 1$ while for $p_{[1]} \to 1$ we have $P\{N = n \mid \underset{\sim}{p}\} \to 1$. For $k = 2$ the sampling rule R^* (the conjugate sampling rule $\overline{R}^*$) which is

stationary is optimal in the sense that it minimizes $E\{N|(p_1,p_2)\}$ uniformly in (p_1,p_2) for $p_1 + p_2 > 1$ ($p_1 + p_2 < 1$) among all sampling rules which use (S^*,T^*) and which take no more than n observations from either population; R^* has additional optimal properties for $k = 2$. The procedure (R^*,S^*,T^*) is generalized for $k > 2$ to accommodate such goals as "Selecting the s $(1 \leq s \leq k-1)$ "best" Bernoulli populations with regard to order," and is shown to have desirable properties for these goals as well. Some conjectures are made concerning the optimality of (R^*,S^*,T^*) for $k > 2$. The performance of (R^*,S^*,T^*) is compared for $k \geq 2$ with that of other sequential selection procedures that have been proposed in the literature. An extensive bibliography is included.

The Principal Investigator has submitted a research proposal to ARO-D to pursue new investigations in this and related areas, and to further exploit research findings already obtained. At this time the Principal Investigator would like to express his thanks to the Army Research Office - Durham for its strong support of this research activity. He is indebted to Mr. Ralph E. Shear of the BRL for having brought to his attention the interesting and important practical problem that led to the present studies, and to Dr. Robert Launer of ARO-D for his constant encouragement.



Summary of Research Completed on the Contract

Technical reports, the research for which was supported in whole or in part by ARO Contract DAAG29-77-C-0003.

(These reports were completed after the termination of DAAG29-77-C-0003, but before the start of DAAG29-80-C-0036. Although they were credited to the former contract, they are not listed in the FINAL REPORT on that contract dated September 20, 1978).

Bechhofer, R.E. and Santner, T.J.: "A note on the lower bound for the P{CS} of Gupta's subset selection procedures," TR 401, January 1979.

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (I): general theory," TR 414, March 1979.

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (II): optimal designs for $p = 2(1)6$, $k = 2$ and $p = 3$, $k = 3$," TR 425, May 1979.

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (III): optimal designs for $p = 4$, $k = 3$ and $p = 5$, $k = 3$," TR 436, October 1979.

Technical reports, the research for which was supported in whole or in part by ARO Contract DAAG29-80-C-0036

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (IV): optimal designs for $p = 4$, $k = 4$," TR 440, January 1980.

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (V): optimal designs for $p = 6$, $k = 3$," TR 441, June 1980.

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (VI); conjectured minimal complete class of generator designs for $p = 5$, $k = 4$ and $p = 6$, $k = 4$," TR 453, April 1980.

McCulloch, C.E.: "Conditions under which $E\{N_1\} = \infty$ for Tong's adaptive solution to ranking and selection problems," TR 480, September 1980.

Bechhofer, R.E. and Tamhane, A.C.: "Tables of optimal allocation of observations for comparing treatments with a control," TR 489, January 1981.

Bechhofer, R.E. and Dunnett, C.W.: "Multiple comparisons for orthogonal contrasts: examples and tables," TR 495, March 1981.

Bechhofer, R.E. and Kulkarni, R.V.: Closed adaptive sequential procedures for selecting the best of $k \geq 2$ Bernoulli populations," TR 510, July 1981.

Faltin, F.W. and McCulloch, C.E.: "On the small-sample properties of the Olkin-Sobel-Tong estimator of the probability of correct selection," Florida State University statistics report M-581, July 1981 (based on research done while the authors were at Cornell University).

Research supported in whole or in part by ARO Contract DAAG29-80-C-0036 (or by DAAG29-77-C-0003 and not listed in the FINAL REPORT on that contract dated September 20, 1978).

Papers (published or accepted for publication)

Barton, R.R. and Turnbull, B.W.: "A survey of covariance models for censored life data with an application to recidivism analysis," Communications in Statistics - - Theory and Methods, A8 (8), 1979, 723-750.

Bechhofer, R.E.: Invited review of Selecting and Ordering Populations - - A New Statistical Methodology by J. Gibbons, I. Olkin and M. Sobel (John Wiley 1977), Journal of the American Statistical Association, 75 (1980), 751-756.

Bechhofer, R.E. and Kulkarni, R.V.: "Closed adaptive sequential procedures for selecting the best of $k \geq 2$ Bernoulli populations." To appear in the Proceedings of the Third Purdue Symposium on Statistical Decision Theory, held at Purdue University, June 1-5, 1981.

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control: general theory." Technometrics, 23 (1981), 45-57.

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (II): optimal designs for $p = 2(1)6$, $k = 2$ and $p = 3$, $k = 3$." Accepted for publication in Sankhyā, B.

Faltin, F.W.: "Performance of the Sobel-Tong estimator of the probability of correct selection achieved by Bechhofer's single-stage procedure for the normal means problem," Abstract 80t-59, Bulletin of the Institute of Mathematical Statistics, 9 (1980), 180.

Faltin, F.W.: "A quantile unbiased estimator of the probability of correct selection achieved by Bechhofer's single-stage procedure for the two population normal means problem, Abstract 80t-60, Bulletin of the Institute of Mathematical Statistics, 9 (1980), 180-181.

Hooper, J.H. and Santner, T.J.: "Design of experiments for selection from ordered families of distributions," Annals of Statistics, 7 (1979), 615-643.

McCulloch, E.E.: "Conditions under which $E\{N_1\} = \infty$ for Tong's adaptive solution to ranking and selection problems." Accepted for publication in Communications in Statistics - - Theory and Methods.

Santner, T.J.: "Designing two-factor experiments for selecting interactions," Journal of Statistical Planning and Inference 5 (1981), 45-55."

Santner, T.J. and Snell, M.J.: "Small-sample confidence intervals for $p_1 - p_2$ and p_1/p_2 in 2 x 2 contingency tables," Journal of the American Statistical Association, 75 (1980), 386-394.

Tamhane, A.C. and Bechhofer, R.E.: "A two-stage minimax procedure with screening for selecting the largest normal mean (II): an improved PCS lower bound and associated tables," Communications in Statistics - - Theory and Methods A8 (4), 1979, 337-358.

Papers submitted for publication

Bechhofer, R.E., and Dunnett, C.W.: "Multiple comparisons for orthogonal contrasts: examples and tables."

Bechhofer, R.E., and Tamhane, A.C.: "Tables of optimal allocations of observations for comparing treatments with a control."

Faltin, F.W. and McCulloch, C.E.: "On the small sample properties of the Olkin-Sobel-Tong estimator of the probability of correct selection."

Graduate students supported by Contract DAAG29-80-C-0036

Fred W. Faltin Radhika V. Kulkarni

Consultant supported by Contract DAAG29-80-C-0036

Ajit C. Tamhane

Ph.D. dissertation and M.S. thesis, the research for which was supported in whole or in part by ARO Contract DAAG29-80-C-0036.

Faltin, Frederick W.: (M.S. thesis) "Estimating the probability of correct selection achieved by Bechhofer's single-stage procedure for the normal means problem." May 1980.

Kulkarni, Radhika V.: (Ph.D. dissertation) "Closed adaptive sequential procedures for selecting the best of $k \geq 2$ Bernoulli populations." August 1981.

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This is the final report on Contract DAAG29-80-C-0036 entitled "Multiple-Decision Ranking and Selection Procedures." The report lists the technical reports and published papers (and those accepted for or submitted for publication) which were written with the full or partial support of the contract. Also included is a brief review of recent significant research progress on the contract.

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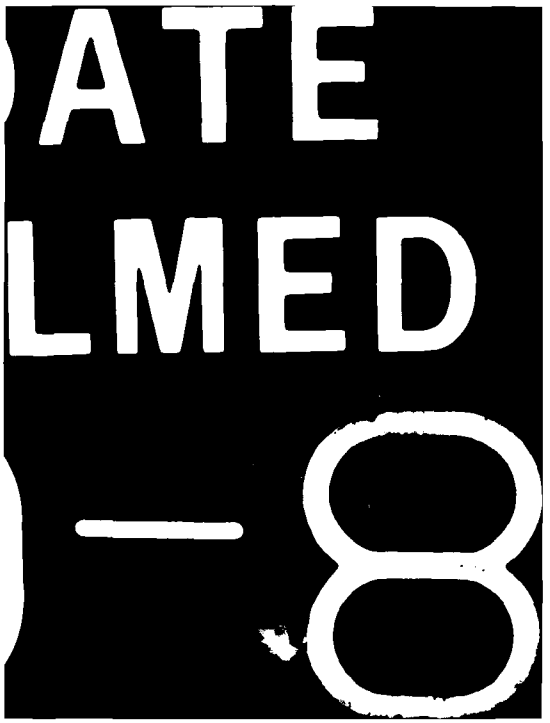
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